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## **CLAIMS**

What is claimed is:

- A method of embedding data into an image, comprising:
   producing an extended image using a halftoning algorithm on a
   collection of images; and
   projecting said extended image onto its first coordinates.
  - 2. The method of claim 1, wherein auxiliary data is embedded into said image imperceptibly, such that data is embedded into said image, and uses halftoning to ensure that a resultant image looks substantially similar to the original image.
  - 3. The method of claim 1, wherein a space of said extended image comprises a Cartesian product of several image spaces.
  - 4. The method of claim 1, wherein said image is represented as matrices of vectors, such that said image M is represented by a n by m matrix M(i,j) of d-dimensional vectors, each pixel M(i,j) of said image is a d-dimensional vector, where d denotes a dimension of a color space.

- 5. The method of claim 4, wherein each d-dimensional vector is in the set {0,1,..., 255}<sup>d</sup>.
- 6. The method of claim 1, wherein said image comprises a source image  $M_0$ , said method further comprising embedding auxiliary images  $M_1$ , ...  $M_k$ , into  $M_0$  imperceptibly.
  - 7. The method of claim 6, wherein said embedding comprises: selecting a set C of extended colors, wherein each member of the set C is a (k+1)-tuple of d-dimensional vectors, and wherein for a member D of the set C, each of the k+1 d-dimensional vectors of D is termed a "coordinate" of D where for  $D = (c_1, c_2, ..., c_{k+1})$  where each  $c_i$  is a d-dimensional vector, the first, second, ..., (k+1)-th coordinates of D are  $c_1, c_2, ..., c_{k+1}$ , respectively.
  - 8. The method of claim 7, wherein the set C is chosen such that for each j in the set  $\{0,1,...,255\}^d$ , there exists at most one member of C such that the first coordinate is j.
- 9. The method of claim 8, wherein the set C is of the same size as {0,1,..., 255}<sup>d</sup>, such that for each j in the set {0,1,..., 255}<sup>d</sup>, there exists exactly one member of C such that the first coordinate is j and the projection of C into its first coordinate results in a bijection.
  - 10. The method of claim 9, further comprising:

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for each pixel location (i,j), selecting said extended output image ExOut as a matrix of elements of C; and generating an embedded source image  $M_0$ ' by taking a first coordinate of the entries of ExOut.

5 11. The method of claim 10, further comprising: extracting the embedded images from  $M_0$ '.

coordinates of the extended output image ExOut.

12. The method of claim 11, wherein said extracting comprises:for each of the pixels M'(i,j) of M<sub>0</sub>', finding the element c(i,j) in the setC such that M'(i,j) is the first coordinate; and

generating reconstructed embedded images  $M_1$ ',  $M_2$ ', ...  $M_k$ ' by setting  $M_u$ '(i,j) equal to the (u+1)-th coordinate of c(i,j), u=1,...,k, wherein the embedded images  $M_0$ ',  $M_1$ ',  $M_2$ ', ...  $M_k$ ' form the

13. The method of claim 12, further comprising: ensuring that the images  $M_0$ ',  $M_1$ ',  $M_2$ ', ...  $M_k$ ' resemble  $M_0$ ,  $M_1$ ,  $M_2$ , ...  $M_k$ 

14. The method of claim 13, wherein an entire image is selectively embedded into the source image, and the embedded image is used for identifying a tampering of said source image.

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- 15. The method of claim 13, wherein an entire image is selectively embedded into the source image, and the embedded image is used for localizing a tampering of said source image.
- 16. The method of claim 13, wherein an entire image is selectively embedded into the source image, and the embedded image is used for reversing tampering of said source image.
  - 17. The method of claim 13, wherein an entire image is selectively embedded into the source image, and the embedded image is used for authentication of said source image.
- 18. The method of claim 1, wherein separate error diffusion algorithms are applied jointly to the embedded images and the source image, such that a choice of output colors is optimized.
  - 19. The method of claim 7, wherein the set C is selectively chosen to be any set of the form (a,f(a)) such that the function f is chosen depending on an application of said method.
  - 20. A method of authenticating an image, comprising: producing an extended image using a halftoning algorithm on a collection of images;

projecting said extended image onto its first coordinates; and

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authenticating said source image by extracting an embedded image from said projection to compare said embedded image to said source image.

- 21. The method of claim 20, wherein, for each member of a set of extended colors C, an output pixel is set to said member at a particular location (i, j) of the extended image to determine whether error is minimized by selecting such a particular output.
- 22. The method of claim 21, wherein an auxiliary image  $M_1$  is generated from the source image  $M_0$  by a permutation of the pixels  $M_1(P(i,j)) = M_0(i,j)$ , where P is a permutation of the pixel locations (i,j).
- 10 23. The method of claim 22, wherein, in the halftoning algorithm, an inverse permutation is applied to restore a spatial relationship before calculating the error function such that a function to minimize is:

$$v_0 \| L(ExOut_0 - M_0) \|^2 + v_1 \| L(PExOut_1 - M_0) \|^2$$

where PExOut<sub>1</sub> is the image defined by:

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$$PExOut_1(i,j) = ExOut_1(P(i,j));$$

24. The method of claim 23, where the parameters  $v_0$  and  $v_1$  are set equal to 1.

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25. The method of claim 23, further comprising:

for each pixel, and for each member d of an extended color set C;

computing an error v(d) at a particular pixel (i,j) by setting the (i,j)-th

pixel of the extended output ExOut equal to d;

determining the member d of an extended color set C which minimizes v(d);

setting the (i,j)-th pixel of the extended output ExOut equal to said minimizing d;

repeating the above process if the maximum number of iterations is not reached and the extended output ExOut has changed between the last two consecutive iterations; and

26. The method of claim 25, wherein, at each iteration, the pixels are traversed in a particular order and for each location (i,j), the pixel ExOut (i,j) is chosen from the set C, such that

$$v_0 ||L(ExOut_0 - M_0)||^2 + v_1 ||L(PExOut_1 - M_0)||^2$$

otherwise terminating said process.

is minimized,

wherein a plurality of iterations are made until a local minimum is reached or the maximum number of iterations is reached.

- 27. The method of claim 26, wherein localized changes to the source image result in changes to the embedded images which are spread throughout the image.
- 28. The method of claim 26, wherein a reconstructed embedded image is compared with the source image to check whether significant changes have occurred.
- 29. The method of claim 26, wherein a reconstructed embedded image is compared with the source image to localize where such changes or corruptions have occurred.
- 10 30. The method of claim 20, further comprising:

  repairing the source image which has been changed by using the embedded image.
  - 31. The method of claim 30, wherein said repairing comprises: setting the embedded image M<sub>1</sub> to be the same as the original source image but after having been permuted by re-shuffling of the pixels thereof; extracting the embedded image image M<sub>1</sub>' from the modified M<sub>0</sub>'; and applying an inverse permutation to M<sub>1</sub>' to obtain a reconstructed image.
    - 32. The method of claim 31, further comprising:

low pass filtering a difference between the image  $M_0$ ' and the image  $M_1$ ' after the inverse permutation, and finding the pixels with predetermined large norms, such that an estimate of where the modification occurs is produced,

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wherein the estimate is used to determine where the embedded image should be repaired such that if a pixel is modified in the source image, then said pixel after permutation is not recovered in the embedded image PM<sub>1</sub>', but is interpolated from other pixels which are determined not to have been modified.

- 33. The method of claim 32, further comprising:

  processing said estimate with a morphological image processing operation.
- 34. The method of claim 20, further comprising:

  detecting whether the image has been tampered with.
- 15 35. The method of claim 34, further comprising:

  repairing portions of the image which have been detected as being tampered with.
  - 36. The method of claim 20, wherein said image has a hologram property such that for an image having modified and unmodified portions of an original

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of the image, an unmodified portion of an original of the image is used to reconstruct the image.

37. The method of claim 20, further comprising:

in an image having a portion with one of modified and defective pixels, taking a portion of said image which has not been modified and reconstructing the entire image.

- 38. The method of claim 34, further comprising: self-repairing said image with a portion of an original of said image.
- 39. The method of claim 20, further comprising: localizing any tampering of said image; and detecting said tampering.
- 40. A method of processing an image, comprising:
  scrambling a source image to create a scrambled version of the source image;
- embedding the scrambled version of the source image into the source image;

extracting the scrambled version of the image; and unscrambling the extracted scrambled image to produce an extracted unscrambled image.

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- 41. The method of claim 40, further comprising:

  comparing the extracted unscrambled image to said embedded image,
  thereby to localize any tampering with the image.
- 42. The method of claim 41, further comprising: repairing a tampered-with portion of the image.
- 43. A method for embedding data into an image, comprising:

  calculating errors corresponding to a halftone extraction algorithm

  applied to a plurality of images, to select an extended image such that a sum of the errors is minimized; and

  projecting said extended image onto its first coordinates.
- 44. A system for embedding data into an image, comprising:

  an error calculator for calculating errors corresponding to a halftone extraction algorithm applied to a plurality of images, to select an extended image such that a sum of the errors is minimized; and
- a projection unit for projecting said extended image onto its first coordinates.
- 45. A signal-bearing medium tangibly embodying a program of machinereadable instructions executable by a digital processing apparatus to perform a method for embedding data into an image, said method comprising:

producing an extended image using a halftoning algorithm on a collection of images; and

projecting said extended image onto its first coordinates.